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Freeze out of WIMP's and WIMP Miracle:

So far we have considered the relic abundance of massive neutrinos in the cases that $m_\nu \ll 1\,MeV$ and $1\,MeV \ll m_\nu \ll m_{Z,W^\pm}$.

Now let us consider a generic Weakly Interacting Massive Particle (WIMP), denoted by X that has a mass m_X. Its interactions have a strength similar to that of weak interactions. It therefore behaves similar to a neutrino if $m_X \ll m_{Z,W^\pm}$, which we considered before.

We are interested in finding the relic density of X if $m_X \gg 100$ GeV. Starting in thermal equilibrium at $T \gg m_X$, the situation will be similar to that for a neutrino with $m_\nu \ll 1\,MeV$. Once T drop below m_χ, then χ's annihilate to lighter particles and n_χ follows its equilibrium value for a non-relativistic species. Eventually, annihilation rate becomes smaller than the Hubble expansion rate, at which time freeze out happens. The comoving number density of χ, or $\frac{n_\chi}{s}$, will remain constant from then on.

For a generic WIMP we have:

$\Gamma_{ann} = n_\chi \langle \sigma_{ann} v_{rel} \rangle$

However, for $m_\chi \gg m_{Z,W^\pm}$ we have:

$$\langle \sigma_{ann} v_{rel} \rangle \sim \frac{\alpha_\omega^2}{m_\chi^2} (\underbrace{a}_{s\text{-wave}} + \underbrace{b \langle v^2 \rangle}_{p\text{-wave}} + \dots)$$

Here m_χ^{-1} determines the range of annihilation interactions, while for $m_\chi \ll m_{Z,W^\pm}$ it is given by $\frac{m_\chi}{m_{Z,W^\pm}^2}$. Note that $G_F \sim \frac{\alpha_W}{m_{Z,W^\pm}^2}$, where $\alpha_W \sim O(10^{-3})$ (α_ω is a gauge fine structure constant).

We can now find freeze out temperature $T_{f.o.}$ by equating

Γ_{ann} and H (as we did before). For simplicity, we focus on the case where $a \neq 0$, hence annihilation in the S-wave dominates;

$$\Gamma_{ann} \simeq H \Rightarrow \frac{\alpha_\omega^2}{m_\chi^2}\left(\frac{T_{f.o.}\, m_\chi}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\chi}{T}\right) \simeq \left(\frac{\pi^2}{90} g_*\right)^{1/2} \frac{T_{f.o.}^2}{M_P}$$

$$\Rightarrow \exp\left(\frac{m_\chi}{T_{f.o.}}\right) \simeq (2\pi)^{-3/2}\left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \alpha_\omega^2 \left(\frac{m_\chi}{T_{f.o.}}\right)^{1/2}\left(\frac{M_P}{m_\chi}\right)$$

$$\Rightarrow \frac{m_\chi}{T_{f.o.}} \simeq 25 - \frac{1}{2}\ln\left(\frac{\pi^2}{90} g_*\right) + \frac{1}{2}\ln\left(\frac{m_\chi}{T_{f.o.}}\right) + \ln\left(\frac{100\ \text{GeV}}{m_\chi}\right)$$

$$\boxed{\Rightarrow \frac{m_\chi}{T_{f.o.}} \sim 25 + (\text{logarithmic corrections})}$$

Again, we find a robust expression for $T_{f.o.}$. Repeating what we did in the previous lecture, we find:

$$\frac{n_\chi}{s} = \frac{n_\chi^{f.o.}}{s_{f.o.}} \simeq \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \frac{25\, m_\chi}{4 M_P\, \alpha_\omega^2}$$

Since the contribution from χ to the energy density cannot exceed that from the dark matter (which is ~ 6 times that from baryons), we have,

$$\frac{n_\chi}{s} m_\chi \leq 6 \frac{n_B}{s} \times (1\,\text{GeV}) \Rightarrow \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \frac{25}{4\alpha_\omega^2} \frac{m_\chi^2}{M_P}$$

$$\leq 6 \times 9 \times 10^{-11} \times (1\,\text{GeV})$$

Note that the inequality is saturated if χ accounts for all of the dark matter in the universe.

It is seen that for $m_\chi \simeq 100$ GeV we have $\rho_\chi \sim 0.05\,\rho_{DM}$, while for $m_\chi = 1$ TeV we have $\rho_\chi \sim 5\,\rho_{DM}$.

Therefore within the WIMP mass window of $(10^2 - 10^3)$ GeV the contribution from WIMPs is in the ballpark area of dark matter.

It is remarkable that this mass window is the same as the one where commonly believed there is new physics beyond the standard model. Many extensions of the standard model including new particles of weak strength interactions and masses in the 100 GeV – 1 TeV window have been proposed. This window is within the energy reach of colliders, such as the Large Hadron Collider, which is built to discover new physics in this energy range.

The coincidence of the WIMP mass range where it can account for dark matter, and that motivated from particle physics, is called the WIMP miracle.

This tells us that there is something special about particles with weak strength interactions and mass in the 100 GeV – 1 TeV window. For this reason, it is widely believed that such new particles indeed exist.

Chemical Freeze out vs Kinetic Freeze out:

What happens at $T_{f.o.} \sim \frac{m_X}{25}$ is actually chemical freeze out. This means that the comoving number density of WIMPs remains constant for $T < T_{f.o.}$. Note that for $m_X \sim 100\,\text{GeV} - 1\,\text{TeV}$ the freeze out temperature is very high, $T_{f.o.} \sim 4\,\text{GeV} - 40\,\text{GeV} \gg 1\,\text{MeV}$.

Even though X annihilation is inefficient at $T < T_{f.o.}$, it can interact with particles that have weak interactions (ν's, $e^{\pm}$, ...) effectively down to $T \sim 1\,\text{MeV}$ (because of the efficiency of weak interactions until then).

These interactions keep X in kinetic equilibrium with the plasma. Since X's are non-relativistic at this time, then:

$$\frac{1}{2} m_X \langle v_X^2 \rangle = \frac{3}{2} T \Rightarrow \boxed{\langle v_X^2 \rangle = \frac{3T}{m_X}}$$

The r.m.s velocity of χ is therefore redshifted according to;

$v_{r.m.s} \propto a^{-1} \qquad T \gtrsim 1\,MeV$

Finally, at $T < 1\,MeV$, weak interactions freeze out. At this time χ's completely decouple from the plasma. They move freely with an initial velocity given by a Gaussian distribution with $v_{r.m.s} \sim \sqrt{3 \times \left(\frac{1\,MeV}{m_\chi}\right)}$.

The velocity of each χ particle is redshifted $\propto a^{-1}$, as expected for a free particle.

Next time we will see what happens between the kinetic decoupling of WIMPs and the start of growth in density inhomogeneities because of gravitational clumping.

In passing we note that a WIMP decouples from the plasma much earlier than baryons

(~1 sec vs ~400,000 year). This implies that inhomogeneities in dark matter grow more than that in baryons. By the time baryons decouple, there is already gravitational wells from dark matter in which baryons fall. This plays an important role in the growth of density fluctuations and subsequent formation of the structure.